MCAMC

Individual Round

Middlesex County Academy Math Competition

April 13, 2024

This section of the competition consists of **25 questions** to be completed **individually** within **1 hour**.

Questions are **not necessarily** in order of increasing difficulty.

No calculators, notes, compasses, smartphones, smartwatches, or **any other aids** are allowed.

All answers must be written **legibly** on the answer sheet to receive credit.

All answers are **non-negative integers** between 0 and 999 **inclusive**.

There is **no need to include units** for any answer, and the units are always assumed to be the units in the question.

Best of luck!

1 Answer Sheet

Name: ______________________________

Individual ID: ______________________________

Team Name: ______________________________

Team ID: ______________________________

Please write your answers on this sheet **legibly**. Follow the rules outlined on the first page.

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________
13. ____________
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18. ____________
19. ____________
20. ____________
21. ____________
22. ____________
23. ____________
24. ____________
25. ____________

2 Problems

1. Find the value of $1 \times 2 \times 3 + 4 \times 5$.

2. Alfred sharpens 8 pencils per minute, and Ben takes 30 seconds to sharpen a pencil. How many times faster does Alfred sharpen pencils than Ben?

3. The ratio of perimeters between two similar triangles is 3. What the ratio of their areas?

4. A two digit positive integer is called a **palindrome** if it can be written the same way backwards and forwards. For example, 33 is a palindrome, but 30 is not because $30 \neq 03 = 3$. Find the number of two-digit palindromes.

5. Aaron has a fair coin, which he flips 4 times. If the probability that he gets more heads than tails is $\frac{a}{b}$, where a, b are positive integers and $\frac{a}{b}$ is in reduced form, find $a + b$.

6. What is the square of the diameter of a sphere circumscribed around a unit cube?

7. There are three light bulbs. The first light shines for 3 seconds, and is off for 3 seconds. The second light shines for 4 seconds, and is off for 4 seconds. The third light shines for 5 seconds, and is off for 5 seconds. In how many seconds will all three lights turn on at the same time?

8. Points A, B, C, and D are randomly chosen on a circle such that they form a square. The probability that chords $\overline{AB}$ and $\overline{CD}$ intersect can be expressed by the fraction $\frac{X}{Y}$ in simplest form. Find $X + Y$.

9. A robot mouse is programmed to take 1 inch strides either north or east. However, due to energy constraints, the mouse can only move north 6 times and east 4 times. How many unique paths can the mouse take if it takes as many steps as possible?

10. A magic fountain appears and asks you to toss in coins. Luckily, you brought an unlimited supply of pennies. After you toss 10 coins into the magic fountain, the fountain then gives you back one coin. If you toss another 10 coins into the fountain, the fountain gives back two coins. For each successive 10 coin donation, the fountain returns one more coin than it did previously. How many coins will you have to toss before you break even?

11. How many ways can books A, B, C, D, E, F, and G be ordered on a bookshelf if books A, E, and F have to be next to each other in that specific order?

12. Consider a series of 40 nested circles where the innermost circle has radius 1, the second smallest circle has radius 2, the third smallest has radius 3, etc. The space between nested circles is randomly colored either red, green, or blue. Starting and ending on the circumference of the outermost circle, if you travel across its diameter, what is the expected number of times you will cross a boundary between two colors?

13. Assume that $2a + b = c$. The digit in the hundreds place of c is the same as the digit in the tens place of b, the tens place of c is a 0, and b and c share the same ones digit. If b and $a + b$ are two-digit numbers, what is the value of a?

14. A vertically oriented parabola has vertex (6, 6) and one zero at the origin. Find the area of the triangle formed by the vertex and zeroes of the parabola.

15. 1200 students were given a 3-question multiple choice test and the sum of all their scores was 2024. What is the maximum median score of all the students if all the students answered at least 1 question right?

16. Ishaan has 5 black marbles and 8 white marbles. If he chooses two marbles at random without replacement, the probability they will be different colors can be expressed as $\frac{A}{B}$, where A and B are relatively prime. Find $A \times B$.

17. The infinite sum $2 + \frac{1}{2+\frac{1}{2+\dots}}$ converges to a finite value that can be written as $a + \sqrt{b}$ for integers a and b. Find $a + b$.

18. Beth bakes a large number of macaroons. If she packages them in rows of 6, she has 5 macaroons left over. If she packages them in rows of 7, she has 4 macaroons left over. If she packages them in rows of 8, she has only 1 macaroon left over. What is the least number of macaroons she could have baked?

19. Let $s(n)$ denote the sum of the digits of a number n. Find the number of three-digit positive integers n for which some element of the sequence $n, s(n), s(s(n)), \dots$ is 2.

20. A quarter is placed on a stationary circle that has a radius twice the size of its own. When the quarter completes one revolution around the circle, how many revolutions has the quarter itself made?

21. Aditya, Ben, and Charlie walk on concentric circular tracks of radii 5m, 10m, and 15m respectively. Ben walks twice as fast as Aditya, and Charlie walks twice as fast as Ben. If they all line up along the diameter of Charlie's track every half hour, how long (in minutes) would it take Charlie to walk 20 times around Aditya's track?

22. At the MCAMC Cafe, cookies are sold in boxes of 7 or 11. Nathaniel, being the little trickster he is, wants to order the largest number of cookies that cannot be served in boxes of 7, 11, or a combination of both. Since the MCAMC Cafe's number one priority is not to make the customer upset, they will not deny Nathaniel his offer. What is the greatest number of cookies Nathaniel can order to meet this goal?

23. Square $ABCD$ has a side length of 2. Equilateral triangles $\triangle ABE$ and $\triangle CDF$ are drawn such that E and F lay inside $ABCD$. If the portion of the area of $ABCD$ not covered by these triangles can be expressed as $\frac{a\sqrt{3}}{b}$, find $a + b$.

24. Four circles of equal radii are inscribed inside a unit circle such that each circle is tangent to the unit circle and its 2 neighbors for a total of 3 tangent points. Another circle is then drawn such that it is tangent to the four inscribed circles and is not another unit circle. If the area of this circle can be expressed as $(a - b\sqrt{2})\pi$ for integers a and b, what is the value of $a + b$?

25. An isosceles triangle has a height of 80 in and base of 120 in. A circle is inscribed inside of it such that it is tangent to all three sides of the triangle. A circle is then drawn such that it is tangent to the previous circle and the two sides of the triangle that are not the base. This process is repeated. The radius of the fourth such circle can be expressed as $\frac{a}{b}$. Find $a + b$.