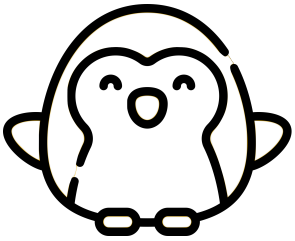


MCAMC

2022

Live Round

Middlesex County Academy

April 30, 2022

This section of the competition is to be completed by **your team** within **1 hour**. This section consists of **8 sets of 3 questions each**. You will receive each set after you have completed the previous.

No calculators, notes, compasses, smartphones, smartwatches, or **any other aids** are allowed.

All answers must be written **legibly** on the answer sheet to receive credit.

Answers must be **exact** (do not approximate π) and in **simplest form**, with all fractions expressed as improper fractions.

Examples of **unacceptable answers** include: $\frac{4}{6}$, $1\frac{1}{3}$, $3+2$.

Examples of **acceptable answers** include: $\frac{2}{3}$, $\frac{4}{3}$, 5.

There is **no need to include units** for any answer, and the units are always assumed to be the units in the question.

Either **exact decimal answers** or **improper fractions** will be accepted (i.e. 0.25 and $\frac{1}{4}$ are both acceptable).

Some questions will require a brief explanation. Additionally, questions may have no answer. If so, the correct response is **None**.

Best of luck!

1 Live Round: Set 1, Dora and Boots

Team Number: _____

1. Dora bought 20 cupcakes for her birthday party. She has 3 guests, plus her best friend Boots. How many cupcakes should be given to each person (including herself) at the party so that each person has an equal number of cupcakes?

2. Boots traces the path that he and Dora must take on his trusty map. The route forms a triangle with perpendicular sides of lengths 5 and 12, but Boots realizes that he can get to the same point with a straight line through the bridge guarded by the Grumpy Old Troll! What is the length of this alternate path?

3. Today is Tuesday, and it is Dora's birthday. If Boot's birthday was 108 days ago, what day of the week was it on? Answer 1 if the answer is Sunday, 2 if the answer is Monday, 3 if the answer is Tuesday, 4 if the answer is Wednesday, 5 if the answer is Thursday, 6 if the answer is Friday, and 7 if the answer is Saturday.

2 Live Round: Set 2, Scooby and Shaggy

Team Number: _____

1. Shaggy plays with 40 colored balls, 10 each are red, blue, green, and orange. He draws the balls out of a box without replacement. If the probability that Shaggy draws two same-colored balls in two turns can be expressed by the fraction $\frac{p}{k}$ in simplest form, find $p + k$.

2. Scooby is drawing socks out of a drawer. The lights are off, however, so he cannot see the color of the socks that he is drawing out. Say that Scooby has many socks of the colors red, purple, blue, and green. What is the minimum number of socks Scooby has to draw in order to guarantee he gets two pairs of matching socks? The same sock cannot be part of two pairs.

3. Scooby and Shaggy love games with mysteries and surprises. Shaggy is in a room with twelve opaque, large boxes. Scooby silently hides in one of the boxes, and hides a surprise in another. Prior to seeking them, Shaggy wants to know the number ways Shaggy and Scooby are located. Assuming it doesn't matter whether Scooby or the surprise is to the left, how many ways are there for Scooby and the surprise to be in 2 boxes?

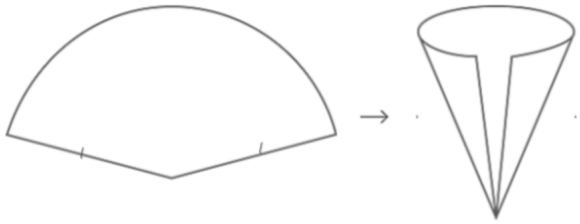
3 Live Round: Set 3, Barbie and Ken

Team Number: _____

1. Barbie and Ken go to a party with 7 people total. 4 are from LA and 3 are from Malibu. All of the people from Malibu know each other, and all of those from LA know each other. The partygoers shake hands one at a time with other people whom they don't know. How many hand shakes occur?

2. Barbie and Ken are bored one day and decide to play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise, from 1 to 12. Both start on point 12. Barbie moves clockwise and Ken, counterclockwise. In every turn of the game, Barbie moves 5 points clockwise and Ken moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take?

3. Ken has a circular piece of cloth with a radius of 20 centimeters, but he wants to make a new hat out of this piece of cloth. He decides to cut a 30° sector out of this piece of cloth and transform it into a cone! If the volume of the cone can be expressed as $\frac{5}{9}\pi(\frac{a}{b})^2\sqrt{c}$, what is $a+b+c$? (An example of a sector to cone transformation is shown below.)



4 Live Round: Set 4, Sesame Street

Team Number: _____

1. Cookie Monster was born on Saturday, November 9, 2002. On what day of the week will Cookie Monster be 706 days old? Answer 1 if the answer is Sunday, 2 if the answer is Monday, 3 if the answer is Tuesday, 4 if the answer is Wednesday, 5 if the answer is Thursday, 6 if the answer is Friday, and 7 if the answer is Saturday.

2. Big Bird had walked half way from home to Grover's house when he realized he was late. He ran the rest of the way to Grover's. He ran 3 times as fast as he walked. Big Bird took 6 minutes to walk half way to Grover's house. How many minutes did it take Big Bird to get from home to Grover's house?

3. Elmo has a collection of 5-pointed, 6-pointed, and 7-pointed stars. In total there are 600 points and 100 stars. Elmo lost all of his 6-pointed stars, leaving him with only 348 points. Originally, how many 5-pointed stars did Elmo have?

5 Live Round: Set 5, Tom and Jerry

Team Number: _____

1. Tom and Jerry are running for the incoming food prize. Tom and Jerry must run laps around a track in directions of their choice, and the number of times they meet each other determines the level of their prize. Given that they start at the same point on the track, and assuming Tom can run at 5 meters per second and Jerry can run at 3 meters per second, what is the greatest number of times they meet by running around a quarter-mile track for 12 minutes, including the time where they start off together? Note that a mile is approximately 1600 meters.

2. The probability that Jerry can evade Tom is 50%. However, the presence of Spike the Bulldog increases Jerry's chances to 80%! Sadly, Spike the Bulldog is only present in the neighbors' yard 20% of the time. There has been a rumor that Tom is being taken out of the house in exactly 100 days. What is the expected number of days that Jerry will evade Tom in these 100 days?

3. The landlord has cut 10 pieces of cheese and wants to give some to Tom, Jerry, Spike, and Toots! However, the landlord wants to give Tom exactly 2 pieces of cheese. How many ways are there to distribute the cheese between Tom, Jerry, Spike, and Toots given that the remaining characters should get at least one piece of cheese each?

6 Live Round: Set 6, Curious George

Team Number: _____

1. The Man in the Yellow Hat defines the function $\phi(n)$ as a function that finds the number of factors of any number n. He gives the following equation on Curious George's final exam for math using the previous definition:

$$2\log_2 x^4 - 1 = \phi(2^{126})$$

The Man in the Yellow Hat tells George that x is in the form a^b, and asks him to find $a + b$. Help Curious George pass his final exam!

2. Curious George adventures to the jungle and finds himself with 29 more monkeys. The 30 total moneys play a game where they designate partners and then enter the jungle. At night, the jungle turns pitch black, and the monkeys must pair up with each other. They do not, however, know who they have paired up with. If the expected value of people who end up pairing up with their partners can be expressed as $\frac{p}{k}$, find p + k.

3. Curious George wants to send a list of numbers that specify the location of bananas to the Man in the Yellow Hat. Any number b whose base b representation of 2022 ends in the digit 2 corresponds to the presence of a banana. How many numbers should be in Curious George's list?

7 Live Round: Set 7, Spongebob Squarepants

Team Number: _____

1. Squidward and Sandy are playing a game. If Squidward can pick a point inside a specific colored area inside of a rectangle $SQUI$, then he wins. Otherwise, Sandy wins. The height of rectangle $SQUI$ is 5, while the width is 40. The colored area is a triangle of width 34 and a height x. Sandy, who wants to win, makes sure that the game is constructed such that Squidward has a 25% chance of winning, and changes x to a fraction in the form $\frac{a}{b}$. What is $a + b$?

2. Sandy has to solve the following system of equations to finish an invention that she's trying to create:

$$4 \log_x y = 2$$
$$\frac{x^2 - y - 1}{y + 1} = y$$

   The positive solution for y is in the form of $\frac{a+\sqrt{b}}{c}$. Find $a + b + c$.

3. Spongebob stands in a right triangle SAN and throws a snowball at Patrick. Patrick is standing in a circle that is 21 feet away from the center of the incircle of $\triangle SAN$. This circle happens to have an external tangent to this incircle, and this external tangent is 29 feet long. Side SA of $\triangle SAN$ is 8 feet long, side AN is 15 feet long, and side SN is 17 feet long. Using this information, Spongebob wants to find the radius of the circle that Patrick is standing in. (Note: The incircle of a triangle is the largest circle that touches all of the sides of the triangle.)

8 Live Round: Set 8, Phineas and Ferb

Team Number: _____

1. Phineas wants to find the smallest absolute value of x that satisfies the equation $\sqrt{1000 + \frac{999}{x}} = 1000 + \frac{999}{x}$. If the absolute value of the answer is fraction $\frac{p}{k}$ in simplest form, find $p + k$.

2. Phineas and Ferb are planning on making three giant springs, which can compress all the down to a height of nearly zero, becoming virtually circles. They are three mutually externally tangent circles all of which have radii of 6. If the area between the three circles can be simplified and expressed as $a\sqrt{b} - c\pi$, where a, b, and c are positive integers, find $a + b + c$.

3. Phineas and Ferb made an out-of-this-world contraption to choose which of his six friends (Buford, Isabella, Baljeet, Irving, Perry, and Candance) are allowed in the new roller-coaster they created! This contraption chooses a random point inside a pyramid. But this is no ordinary pyramid! This square pyramid, with a base length of 10 and an edge (tip of the pyramid to a base vertex) length of 13, has a sphere perfectly inscribed within it. If the friend chooses a point inside of the circle, they are allowed on the new roller coaster! Otherwise, they are not. If the probability that two individuals with the same first initial are the only people on the roller coaster is $\frac{p}{k}$, find $p + k$ (Note: Phineas and Ferb are not going on the roller coaster.)