

MCAMC
2 0 2 2

Individual Round

Middlesex County Academy

April 30, 2022

This section of the competition is to be completed **individually** within **1 hour**. This section consists of **20 questions**.

No calculators, notes, compasses, smartphones, smartwatches, or **any other aids** are allowed.

All answers must be written **legibly** on the answer sheet to receive credit.

Answers must be **exact** (do not approximate π) and in **simplest form**, with all fractions expressed as improper fractions.

Examples of **unacceptable answers** include: $\frac{4}{6}$, $1\frac{1}{3}$, $3 + 2$.

Examples of **acceptable answers** include: $\frac{2}{3}$, $\frac{4}{3}$, 5.

There is **no need to include units** for any answer, and the units are always assumed to be the units in the question.

Either **exact decimal answers** or **improper fractions** will be accepted (i.e. 0.25 and $\frac{1}{4}$ are both acceptable).

Some questions will require a brief explanation. Additionally, questions may have no answer. If so, the correct response is **None**.

Best of luck!

1 Answer Sheet

Name: _____

Team Name: _____

Team Number: _____ Please write your answers on this sheet legibly, and please follow the rules that were found on the first page.

- | | | |
|----------|-----------|-----------|
| 1. _____ | 8. _____ | 15. _____ |
| 2. _____ | 9. _____ | 16. _____ |
| 3. _____ | 10. _____ | 17. _____ |
| 4. _____ | 11. _____ | 18. _____ |
| 5. _____ | 12. _____ | 19. _____ |
| 6. _____ | 13. _____ | 20. _____ |
| 7. _____ | 14. _____ | |

2 Individual Round

1. Loki is trying to deceive Thor with a math question! He asks Thor to solve the following:

$$1 \times 2 + 2 \times 3 - 3 \times 4 + 4 \times 5 + 67$$

What should Thor's answer be?

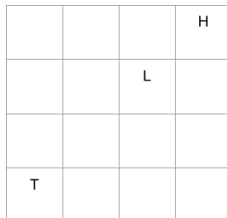
2. Hawkeye can shoot 3 arrows in 1 second! However, once given a boost, he can shoot 5 arrows in 1 second. If Hawkeye shoots for 3 seconds without a boost, but then shoots with a boost for 5 seconds, how many arrows does he shoot?
3. Moon Knight is trying to find the volume of the scarab that leads him to Ammit's tomb! This scarab is in the shape of a cylinder with radius 5 and height 9. What is the volume of the scarab?
4. Thanos can eliminate half a world's population with each snap. He is targeting Asgard, whose population is 800,000. If the probability that Thor has been eliminated by the time Thanos snaps thrice can be expressed by $\frac{a}{b}$ in simplest form, find $a + b$.
5. Dr. Strange is trying to predict Thanos's next move. He is 80% sure that Thanos will attack with his Power Stone. He is unsure about the move after that one, however, as it is later in time. Therefore, he is 20% sure that Thanos will attack with his Reality Stone after his Power Stone. What is the probability that Thanos attacks with his Power Stone, and then his Reality Stone?
6. Hulk is trying to gain the most amount of strength possible in the shortest amount of time. He has three fitness plans, A, B, and C. Plan A is that he will gain 10 XP of strength each day for the entirety of the plan. Plan B is that he will gain the amount of XP that is the square of the day number, i.e. 1 on day 1, 4 on day 2, 9 on day 3 and so on. Plan C is that he will start off at 7 XP a day and each subsequent day the amount of XP he gains increases by 1, i.e. 7 on day 1, 8 on day 2, 9 on day 3, and so on. What is the difference in XP gained between the most effective plan and the least effective plan if he trains for 7 days?
7. Iron Man is trying to make his car as fast as possible. At first, he travels 30 miles at 90 mph. Wanting to increase his speed, he quickly modifies his car, before racing away at 110 mph for another 30 miles. What was his average speed over the trip in miles per hour?
8. Thanos has just escaped and the Avengers are trying to hunt him down. They know that he could have escaped to any one of 1000 planets numbered from 1 - 1000. They also know that he wouldn't have gone to any planet that is a multiple of 7, 11, or 13 due to the harsh environment of those planets. How many planets do the Avengers have to search assuming that Thanos remains at the planet he has escaped to?
9. Tony Stark created a lock to protect the Tesseract! This lock has 5 spaces, where the first 3 spaces are numbers, and the second 2 spaces are letters. There are 2 rules, however. 2 even numbers cannot be next to each other, and 2 vowels cannot be next to each other. How many different combinations are there for the lock?
10. Attempting to track down Taskmaster, Black Widow climbed down a cliff $\frac{5}{8}$ of the time and trekked up a hill $\frac{3}{8}$ of the time. Her climbing rate is 6 miles per hour, and her uphill hiking rate is 2 miles per hour. If the distance that Black Widow climbed is $\frac{p}{q}$ the total distance that she covered, find $p + q$.
11. Doctor Strange has found himself in another infinite loop with Dormammu, the Destroyer of Worlds! Everytime Strange dies in this infinite loop, he continues to come back to life, due to the power of the Time Stone. Every time Strange comes back to life, there is a 25% chance that Dormammu realizes that he is in a time loop at which point he ends the time loop, and a 75% chance that Dormammu doesn't realize that he is in a time loop, and continues through it. What is the probability that the time loop continues for infinite time?

12. Ant Man leaves at 8:30 AM heading for Iron Man from Avengers Headquarters. He flies at a uniform rate of 12 miles per hour. Not knowing that Ant Man was coming to meet him, Iron Man leaves Stark Tower at 9:00 AM heading for Ant Man. He flies at a uniform rate of 16 miles per hour. They both fly on the same 62-mile route between their two starting positions. If the time in the morning that they meet is $ab : cd$, find $1000a + 100b + 10c + d$.
13. Tony Stark has 5 compartments in his mansion in Malibu, which is used to store his Iron Man suits. At the moment, he wants to distribute the Mark I, Mark II, and Mark III suits in these 5 compartments (the compartments are large enough to hold 3 suits). In how many ways can Stark put his Mark I, Mark II, and Mark III suits in these five compartments?
14. Captain America is searching for a criminal and knows that the criminal is in the region bounded by the graph of the equation:

$$| -10x | + | 5y | = 20$$

What is the area that Captain America needs to search assuming that he needs to search the entire region?

15. Thor is stuck in the bottom left corner of a 4×4 grid, and needs to make it to the top right unit square to retrieve his hammer. However, Loki is in the square second from the top and third from the left, and Thor wants to avoid him at all costs since he cannot fight him without his hammer. Assuming he can only move to a square directly above or to the right from his current position each time, how many different paths can Thor take to reach his hammer?



16. Groot, as you may know, is a tree that never stops growing. In particular, it has levels of branches, where each level splits into two branches in the next level. For example, levels 0 - 4 have 1, 2, 4, 8, and 16 branches respectively. If we count each branch separately from the branches it splits into ($1 + 2 + 4 + 8 + 16$ branches up to level 4), what is the the total number of branches that Groot has from level 0 to level 17?
17. Tony Stark has been asked to write the final question for MIT's number theory class, as he is one of the most famous alumni of the institute! He asks: Find the remainder when 23^{13} is divided by the number of factors of 9009.
18. Avengers Headquarters is being attacked by Ebony Maw! The Avengers are protecting their headquarters, which is a cube with side length 300, at its eight vertices. Each of the separate Avengers has an attack range of 50 feet in all directions (including against underground attacks). The volume of space that can be protected by the Avengers with their attacks can be written in the form $a^3 + b\pi$. Find $a + b$. (Note: Because the headquarters is the building being protected, include its volume.)
19. Stark Industries is currently offering internships to promising young students in science and mathematics. On the admissions exam, Bob finds the following question: Find the largest value of the integer k such that $(2022!)^6$ can be divided by 25^k with no remainder.
20. Black Widow loves geometry. Stunned by the lack of difficult geometry problems, she decides to write a particularly complex one. Let ABC be a triangle with $AB = 13$, $AC = 15$, and $BC = 14$, and let circle ω be the circumcircle (circle passing through all 3 vertices) of ABC . Let the angle bisector of $\angle BAC$ intersect ω at a point K other than A . If the cosine of $\angle KBA$ is in the form of $\frac{-a}{\sqrt{b}}$, where a and b are positive integers, find $a + b$.