1 Individual Round

This section of the competition is to be completed **individually** within **1 hour**, and this sections consists of **20 questions**. No aids such as calculators, notes, compasses, smartphones, smartwatches, etc. are allowed. All answers must be submitted on Edulastic in order to receive credit. Answers will all be integers. Examples of unacceptable answers include: 3.5, $\frac{4}{6}$, $\frac{1}{3}$, $3+2$. There is no need to include units for any answer, and the units are always assumed to be the units in the question.

1. Harry challenges Hermione to a battle of brainpower. He tells her to compute

$$1*1+1*2+2*2+2*3+3*3+3*4+4*4+4*5$$

What should Hermione answer?

2. Ron is very hungry and eats 210 chocolate frogs in 3 days! There are 144 calories per frog. How many calories is that per minute?

3. The letters a, b, c, d represent 4 different whole numbers. If $a = 8$, $a * b = a$, $b + c = a$, and $c * (d-1) = a * c$, then what is the value of d?

4. The Sorting Hat has started to give riddles to sort students into their houses! It poses the following question: I can divide 720 without remainder, while I can be divided by the first 6 natural numbers. The sum of my digits is 3, as is the amount of digits. What number am I?

5. 5 Gryffindors stand in a circle. Each sends out a number with their wand from 1-6 uniformly at random. Each sends out a number independent of each other. The probability that the sum of the 5 numbers is 7 is $\frac{m}{n}$ where m and n are relatively prime integers. Find $m+n$.

6. The Hungarian Horntail has been recaptured after the first round! It is restricted to flying in the interior or on the cube. If the Hungarian Horntail can only fly from one vertex to another along a diagonal (including face diagonals), how many different paths (movement from 1 vertex to another regardless of direction) can the Horntail fly on?

7. While he's in the Room of Requirement, Harry finds the design for the Sorting Hat, which is a cone! Harry wants to follow the design and starts of with a circle of radius 8. He then cuts out a quarter of the circle and connects the 2 edges of the $\frac{3}{4}$ circle that remains to form a cone. The outer surface area of the Sorting Hat (which does not have a base) is $a\pi$, where a is a whole number. What is a?

8. The Ministry of Magic's cars are once again being used to transport Harry and the Weasleys to King's Cross! However, the magic they have is dying, and so the tires are slowly wearing out. The tires lose 1 mm of diameter for every 50 km traveled, and the original radius of each of the tires is 300 mm. These cars are traveling at a constant speed of 100 km/h. How much time would the trip to King's Cross need to take for the tires to be at a radius of 250 mm?

9. Professor Vector is fed up with his students. They refuse to do their work or study for the exams, except for Hermione. So, Hermione gets a chance at extra credit! Professor Vector asks the following question: what is the remainder when 3^{13} is divided by 4?

10. In Hagrid's Care of Magical Creatures class, he gives 5 students a Niffler, a cute creature that can seek out treasure. Hagrid has hidden 3 indistinguishable coins within his magical bag that is enchanted by the Extension Charm, making it have incredible space inside. In how many ways can these 3 coins be retrieved by the 5 Nifflers?

11. In Harry's first year, Professor Vector wanted to guard the Sorcerer's Stone using an arithmancy question. For some reason, it was never used. However, it's always a question on the final exam. Here is the question: Professor Vector has his hands on triangles. Each of his triangles has a perimeter of 9, has integer side lengths, and are not congruent to any other triangle. What is the maximum number of triangles he could have?

12. Harry is playing against Ravenclaw! While looking for the Snitch, Harry goes against the wind, which is at an unknown speed "x" MPH. It takes him 15 seconds to travel $\frac{1}{4}$ of a mile. Harry swerves and goes in the opposite direction as he tries to avoid a bludger, and now he goes in the same direction as the wind! Here, it takes him 8 seconds to travel $\frac{1}{4}$ of a mile. In the absence of wind, Harry flies at $\frac{m}{n}$ miles per minute, where m and n are relatively prime positive integers. Find $m+n$.

13. Fred and George have accidentally mixed an infinite deck of exploding cards and an infinite deck normal cards! They decide to play a game of luck with this mixed deck. The rules are as follows:

    (a) The first person to have 3 cards explode in their hand loses.

    (b) On each round, there is a $\frac{1}{3}$ probability that Fred draws one card from the deck, there is a $\frac{1}{3}$ probability that George draws a card from the deck, and a $\frac{1}{3}$ probability that neither of them draw a card and the round goes by with nothing happening.

    The probability that the game lasts at most 3 rounds is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

14. Someone is trying to get into Dumbledore's office, which has a history of having extremely obscure passwords. When looking over Dumbledore's shoulder one day, the intruder only noticed the first and last digit of the 4 digit password. The first digit was 7 and the last digit was 4. He also knows that Dumbledore loves arithmancy, and would make his code divisible by 2,3,4,6,8, and 9. What is the sum of all possible passwords that Dumbledore could have?

15. Harry has just Apparated from 1 location on the coordinate plane to another! He Apparated from $(5,2)$ to $(-2,1)$, which also happens to be a 90 degree counterclockwise rotation about a 3rd coordinate, (a,b), on the coordinate plane. Compute a^2+b^2.

16. Harry is fighting a Hungarian Horntail, and his attacks have a 50% chance of hitting the dragon in its weak spot, otherwise it hits the dragon's scaly flesh. When Harry hits the dragon's weak spot, it takes away $\frac{1}{5}$ of its health, and when he hits the dragon's flesh, it takes away $\frac{1}{10}$ of its health. When the dragon attacks, it takes away $\frac{1}{7}$ of Harry's health. The order of the attacks is always: Harry, Dragon, Harry, Dragon... The probability that the dragon dies before Harry is $\frac{a}{b}$ where a and b are relatively prime positive integers. Find $a+b$. (Note: Harry does not attack the dragon once it is dead.)

17. One of the tasks of the Triwizard Tournament is to find the 2021st term of a certain sequence. Something special about this sequence is that for every positive integer n, the average of the first n terms of the sequence is n. What is the correct answer to this task?

18. Seamus is playing with a 2-by-2 Rubik's cube which is made up of 8 1×1 cubes. Each of the 6 faces of the Rubik's cube are a distinct color. However, one day a Blasting Curse breaks the cube apart into the 8 smaller 1×1 cubes. Seamus wants to put the cube back together placing each of the pieces one by one. Due to how the cube is constructed, he is only able to place a piece if an adjacent piece is already placed (with an exception of the first piece). How many different ways can he choose to put this cube back together so that it is identical to how it was before it was destroyed?

19. Harry and Draco have decided to have another Wizard's Duel, but this time, it's an academic duel! They decide to play a game. They start with three concentric circles of radii 1, 2, and 3 centered at the origin of the complex plane. Harry will first place point A on the circle of radius 1, then Draco will place point B on the circle of radius 2, and finally Harry will place point C on the circle of radius 3. Harry wants to maximize the area of $\triangle ABC$, while Draco wants to minimize the area. If both Harry and Draco play optimally, the area of $\triangle ABC$ is $\frac{m}{n}$ where m and n are relatively prime positive integers. Computer $m+n$.

20. Harry is eating a slice of cheese which is a triangle with side lengths 6, 8, and 10. He is in arithmancy class and has just learned about the properties of triangles. He is trying to find the distance between the center of the inscribed circle and the circumscribed circle of the slice of cheese. The answer to this question can be written in the format $\sqrt{a}$, find a?